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APPLICABILITY OF THE SOUTHWELL PLOT TO THE INTERPRETATION OF TEST DATA FROM INSTABILITY STUDIES OF SHELL BODIES

By

W. H. Horton

F. L. Cundari

March 1969

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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The program was carried out under Contract DA 44-177-AMC-258(T) with Stanford University.

The research was directed toward the development of a better understanding of the fundamental processes in the buckling of shell bodies. The data contained in this report are the result of experimental studies of the instability of shell bodies and the interpretation of the data by means of the Southwell plot.

The report has been reviewed by the U.S. Army Aviation Materiel Laboratories and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

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March 1969

APPLICABILITY OF THE SOUTHWELL PLOT TO THE INTERPRETATION OF TEST DATA FROM INSTABILITY STUDIES OF SHELL BODIES

By

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Prepared by

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ABSTRACT

This report demonstrates, from experimental studies, that the classic critical loads computed from small displacement theory are correct for perfect shells under ideal loading conditions. The method used is an implicit rather than an explicit one. It makes use of the fact that for a realistic structure, the elastic deformation can be associated with the initial displacement from ideal form, the load which produces the motion, and the classic load for the structure by the hyperbolic expression

$$\delta\left(\frac{P_{cr}}{P} - 1\right) = \delta_o$$

By choosing the variables to be δ/P and δ, the relationship can be presented in the form of a straight line whose slope corresponds to the critical load for the ideal case. This is, in essence, the method developed by Aryton and Perry, in 1889, to analyze column data. The generalization to include other structures was foreseen by Southwell in his classic paper of 1932, although he offered no proof. In effect, a general proof exists in the theory of elastic stability as presented by Westergaard in 1922. However, the practical application to shells has, until now, not been made.

By using this technique, the behaviour of cylinders under the action of external pressure, torsion, and axial load and of the instability of spheres and spherical caps under the action of external pressure is examined. In the main, the analyses are conducted on experimental data already published. However, there are notable exceptions, the cylindrical shell under axial load being a case in point.

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FOREWORD

The work reported in this paper was supported by the U. S. Army under Contract DA 44-177-AMC-258(T), U. S. Air Force under Contract AF 49(638) 1495, and the U. S. Navy via a research grant from the U. S. Naval Postgraduate School. Their support is gratefully acknowledged.

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LIST OF SYMBOLS

ARABIC SYMBOLS

a = radius of base of spherical cap

A = amplitude of the displacement

E = Young's modulus

L = length

n = integer 1,2,3

p = pressure

p_{cr} = the critical value of p

P = load

P_{cr} = the prime critical value of p

${}_nP_{cr}$ = the critical value of p corresponding to the n^{th} mode

R = radius

s = circumferential distance

t = distance

w = displacement function

W = elastic displacement amplitude parameter

W' = initial displacement amplitude parameter

x = coordinate direction

GREEK SYMBOLS

$\lambda = \left[a^4(12(1-\mu^2)/(Rt^2\right]^{\frac{1}{4}}$

δ = total elastic deflection

δ_o = initial displacement

μ = Poisson's ratio

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INTRODUCTION

Shell structures are very common. For over a century, engineers have investigated experimentally and discussed theoretically their behavior under various loading conditions. The extent of this endeavor is clear from the fact that some 2339[1,2] books and papers were written in the field prior to 1956. Nevertheless, many facets of the subject are still unresolved. Indeed, it has become traditional to expect wide divergence between experimental results and theoretical predictions when certain problems of stability are considered. This is particularly so for circular cylindrical shells under axial compression and spherical shells under external pressure. The exact causes of the discrepancy are hard to identify. However, one factor of importance is easily recognized. Theories are normally developed for ideal bodies with perfect loading conditions and boundary restraint; whereas tests are always made with realistic structures imperfectly loaded and constrained. Thus, if agreement between theory and experiment is to be achieved, it will be through an implicit rather than an explicit process. The Southwell[3] technique, which has already been demonstrated to have general applicability in questions of elastic stability of plates and columns,[4] is such a process. The present paper establishes the validity of this technique for shell problems.

The analytical basis of the work is older than the Southwell concept itself, being found in the study made by Westergaard[5] in 1922. In this very general analysis, the relationship among elastic deformation, the initial deviation from ideal form, the load level, the critical load and its harmonics was derived. The starting point was the Lagrangian expressed in generalized coordinates. Westergaard considered not only initial geometric variances but also nonhomogeneity and eccentricities of loading. He applied the theory of minimum potential and obtained a general relationship among the actual load (P), the critical load for a particular mode (${}_nP_{cr}$), the amplitude of that component of the initial irregularity which corresponds to the n^{th} mode (${}_n\delta_o$), and the appropriate associated component of elastic deflection (δ).

This relationship is

$$\delta = \frac{P}{{}_nP_{cr} - P}\,{}_n\delta_o \tag{1}$$

Clearly, there are n terms of this type and the total elastic deflection of the structure under the given load system is

$$\delta = \sum_{n=1}^{\infty} \frac{P}{{}_nP_{cr} - P}\,{}_n\delta_o \tag{2}$$

which is, of course, identical to the Southwell formulation for the column.

In many problems, the first mode predominates, and, thus, a single-term expression of the type given in equation (2) adequately describes the behavior. Hence, the relationship between δ and P is hyperbolic. However, if the variables are taken as δ/P and δ, the relationship is linear.

EXPERIMENTAL EVIDENCE

To illustrate the process, we chose the simplest curved structure with the most elementary loading; viz, a circular arch compressed by a point load at its vertex. Langhaar, Boresi, and Carver[6] have examined this problem both theoretically and experimentally. The arch which they used was of 10-inches radius and was made from a strip of aluminum 2.5 in. wide and .031 in. thick. It was simply supported at its ends. The load displacement relationship obtained in the test is given in Figure 1. When this is plotted in the Southwell fashion, the characteristic straight line (Figure 1) is obtained. The critical load determined from this line is 4.12 lbs, which is in excellent agreement with the theoretically predicted value of 4.17 lbs.

A somewhat more complex problem is the behavior of a tube under external pressure (p). This problem has been treated analytically by Sturm[7], and the validity of his solution has been well established by his own tests and those of Cleaver.[8] However, from the viewpoint of this paper, the most important factor in Sturm's analysis lies in his demonstration that for a slightly imperfect cylinder, the initial radial deformation amplitude is related to the subsequent amplitude by the equation

$$\delta = \frac{P\,\delta_o}{P_{cr} - P} \qquad (3)$$

This equation is, of course, identical to the general equation derived by Westergaard and clearly shows that a Southwell representation of test data should be applicable. When the load displacement data given in Figure 16 and 18 of Sturm's paper are replotted in the Southwell form, the straight lines given in Figures 2 and 3 of this report are obtained. The critical loads obtained from the several slopes are in excellent agreement with the theoretical predictions.

Subsequent to the work of Sturm, a number of experimental studies were made on the stability of ring-reinforced circular cylinders under external pressure loading. (In one particular series of tests, the shells failed by general instability.) It is with reference to the interpretation of these tests that the first practical application of the Southwell method to shell bodies is recorded. Galletly and Reynolds[9] made the analysis. In their paper, they noted the applicability of the Southwell process to many problems and suggested a modification for the case in question. This was the use of strains instead of displacements, a step which is readily seen admissable and which is analytically demonstrated in Westergaard's paper. The results which they obtained are most important. For the first test which they report, observations were made at 21 different points on the body. The critical loads derived from these measurements are in excellent agreement with one another. They are listed in Table I.

It is seen that the critical loads range from 168 lb to 178 lb, with a mean of 172.2 lb. These values compare well with the theoretical load of 173 lb computed in accordance with Kendrick's[10] theory.

The second set of data presented by these researchers is a comparison of the

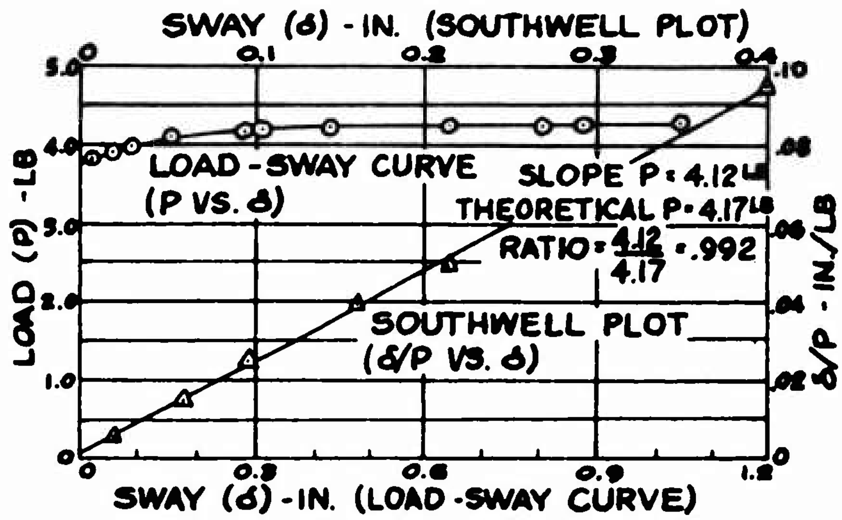

FIGURE 1. Load-Sway Relationship for Elastic Arch and Corresponding Southwell Plot (Ref 6).

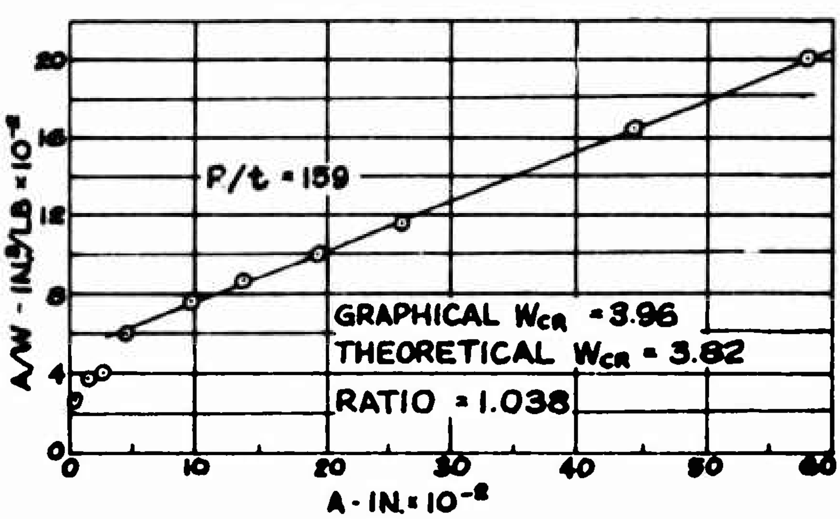

FIGURE 2. Circular Cylindrical Shell under External Pressure (Data from Fig 16, Ref 7).

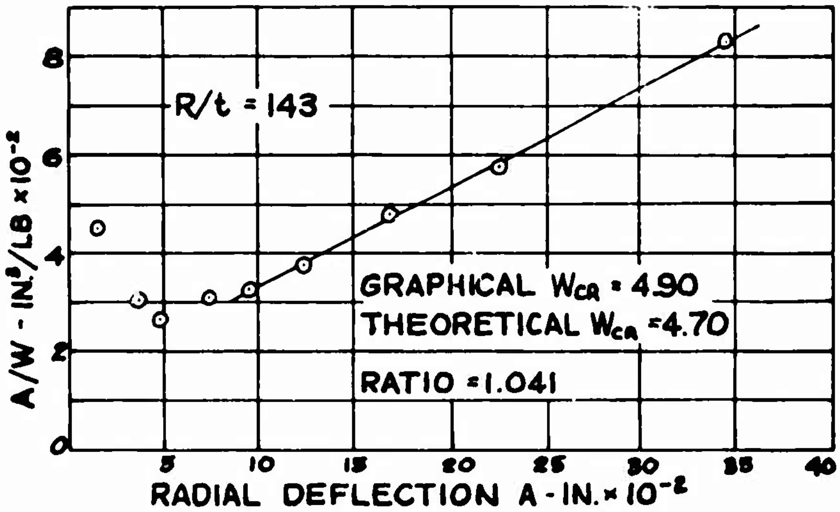

FIGURE 3. Circular Cylindrical Shell Under External Pressure (Data From Figure 18, Ref 7).

TABLE I. COMPARISON OF MULTIOBSERVATIONS ON SINGLE SPECIMEN

Station No.	Southwell Load	Station No.	Southwell Load
1	172	12	171
2	171	13	172
3	173	14	174
4	171	15	173
5	173	16	169
6	169	17	168
7	171	18	174
8	178	19	174
9	174	20	168
10	173	21	174
11	174		

Mean value of load	172.2
Theoretical value	173

Ratio of $\frac{\text{mean value}}{\text{theoretical value}}$ = .995

results of tests made on five similarly constructed shells, whose theoretical collapse pressures are 150 psi and 1000 psi. The ratios of the critical load levels computed from Kendrick's theory and the values deduced from the tests are given in Table II. Again, the comparisons are seen to be very good indeed.

Donnell[11] in his classic paper on the Southwell method appears to have been the first to consider whether or not the process could be applied to the thin-walled circular cylinder under axial load. He examined the problem analytically, taking as his starting point his approximate solution for the instability of a cylinder under axial compression in which he had considered initial deviations from cylindrical shape.

He chose the deflection function to be given by

$$w = W \sin \frac{\pi x}{L} \sin \frac{\pi s}{L} + W(W+2W') \frac{\pi^2 R}{8L^2} \cdot \cos \frac{2\pi x}{L} \qquad (4)$$

The reason for this choice lay in the fact that the second term was most useful in annulling some of the second-order extension with first-order extensions and, thus, was effective in reducing the internal energy. This initial deviation from cylindrical form was taken to be given by

$$w' = \frac{W'}{W} w \qquad (5)$$

From this basis, he computed that the relationship among the load, the critical load, and the initial and subsequent deformations would be given by

$$P = \frac{W}{W' + W} \cdot P_{cr} \frac{1 + \frac{\pi^4 R^2}{4L^4} \cdot (W+2W')(W+W')}{1 + \frac{\pi^4 R^2}{8L^4} \cdot \frac{(W+2W')^2 (2W+W')}{W+W'}} \qquad (6)$$

where P_{cr} is the critical load for the perfect shell and is given by the normal equation

$$P_{cr} = \frac{E.t.}{\sqrt{3(1-\mu^2)}} \cdot \frac{1}{R} \cdot 2\pi R t \qquad (7)$$

It is clear that if Donnell's large displacement derivation is true, there must be some classes of shells for which the multiplying factor on the Southwell form is not unity. But it is apparent that if W and W' are small compared with $2L^2/\pi^2 R$ or when W' is very small compared with W, the Southwell plot should yield accurate results.

In view of Donnell's analytical development and the acute interest in circular cylinders under axial compression, it might be thought that the question would have been experimentally resolved. Unfortunately, at the time that Donnell was studying this problem, it was not technically feasible to use noncontacting displacement transducers to measure the radial movement of the wall of an axially compressed shell model with the degree of accuracy

TABLE II. COMPARISON OF SINGLE OBSERVATIONS ON A RANGE OF SPECIMENS	
Specimen No.	$\frac{\text{Southwell Load}}{\text{Theoretical Load}}$
1	1.011
2	1.022
3	.861
4	1.076
5	.882
Average	.970

required to verify the analysis. Thus, Donnell was constrained to theoretical observation only.

Flügge,[12] in his text on "Stresses in Shells," appears to be the next author to refer application of the Southwell plot to test results on shell bodies. In his discussion, he emphasizes that more than one mode may be involved in the total deformation; thus, since in many shells there exist a number of critical loads immediately above the lowest one, several terms of the displacement series may grow to infinity together. In this detail, then, the cylinder under axial compression differs from the column for which the eigenvalues are significantly separated.

However, he states that if care is taken to measure a displacement which is large for the lowest buckling load but small for all others, then the Southwell technique is applicable. He substantiated his remark by reference to unpublished experiments conducted by Kromm and Flugge in the early 1940's. These tests were made on cylinders under various combinations of axial compression and torsion. Professor Flügge has remarked that Southwell plots were obtained only when the predominant loading was torsional.

In our research, we have found that if well-made thin-walled shells are tested with extreme care to ensure uniformity of load distribution, and if noncontacting probes of high resolution are used to determine the wall motion, load-displacement relationships of the hyperbolic type are obtained. Data for typical tests are given in Figures 4 and 5. Both these observations were made on the same shell. The data have been plotted in the Lundquist[13] manner. The agreement between the classic critical loads and the values computed for these plots is good (maximum error, 1.98%). The cylinder used was made from a thick-walled tube of aluminum by machining and was circular to within 1/1,000 in. and was uniform in thickness to better than 1/10,000 in. It had an R/t of 335 and an L/D of 1.87. The test was made in which loading was via a thermal ram.[14] Deflection measurements were made using a Fotonic sensor, an electro-optical measuring device of high resolution.[15] The test is fully described in reference 16.

In the case of torsional loading, it is much easier to determine experimentally the wall motions during instability. Consequentially, it is relatively easy to obtain good results in this case. A typical set of data, obtained from a test[17] on an orthotropic cylinder, is given in Figure 6.

The literature contains much material with reference to the instability of spheres and spherical caps under the action of normal point loads, uniform external pressure and combinations thereof. This has been examined in Reference 18.

The experimental work of Ashwell[19] and Evan-Ivanowski, Cheng, and Loo[20] on spherical caps subjected to a concentrated load at the apex provides several cases in which the load-displacement curves can be successfully analyzed by the Southwell method. The load-displacement curve and the corresponding Southwell line are given in Figure 7 for Ashwell's test on a

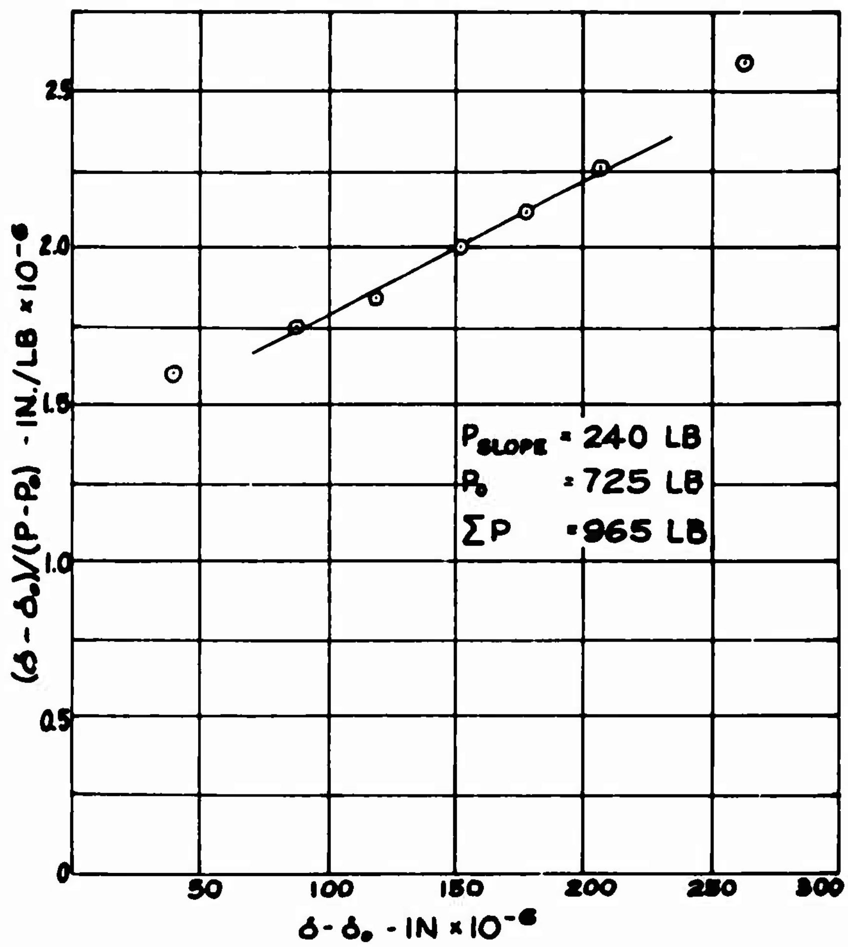

FIGURE 4. Lundquist-Type Plot for Thin-Walled Circular Cylinder under Axial Compression Observation Point No 1.

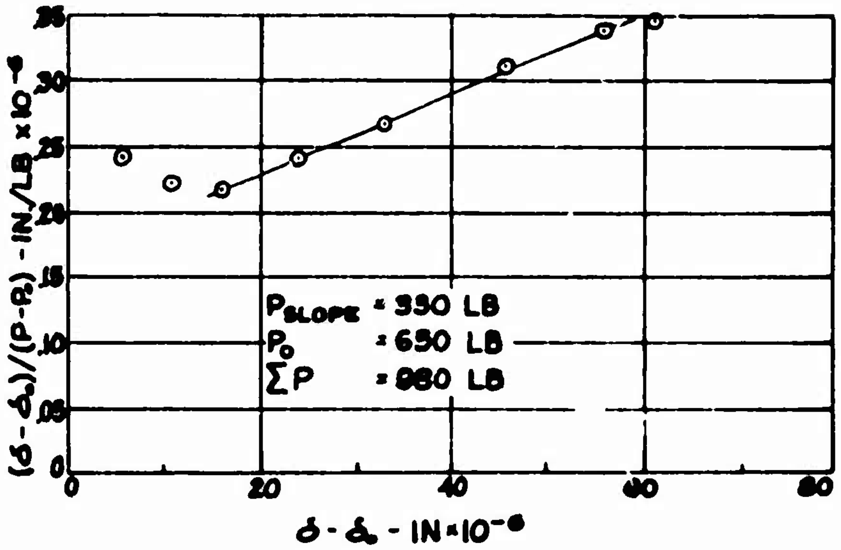

FIGURE 5. Lundquist-Type Plot for Thin-Walled Circular Cylinder Under Axial Compression Observation Point No 2.

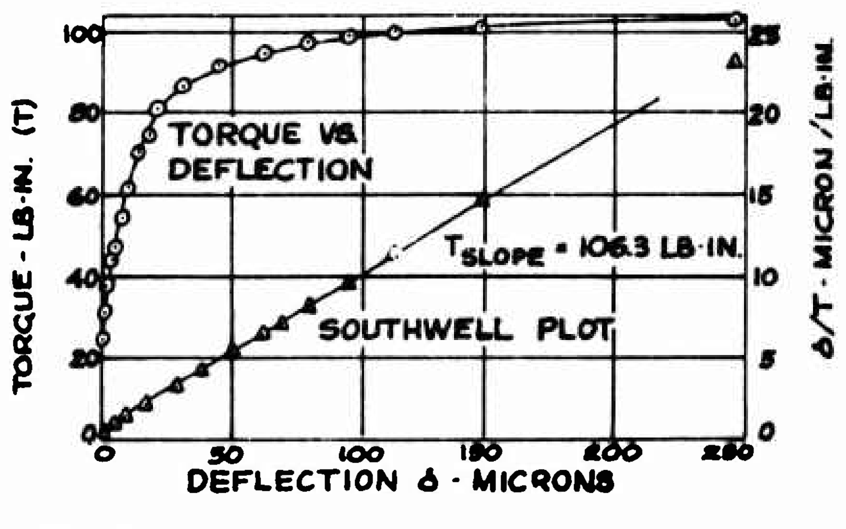

FIGURE 6. Torsion Versus Inward Wall Motion and Corresponding Southwell Plot for Orthotropic Cylinder.

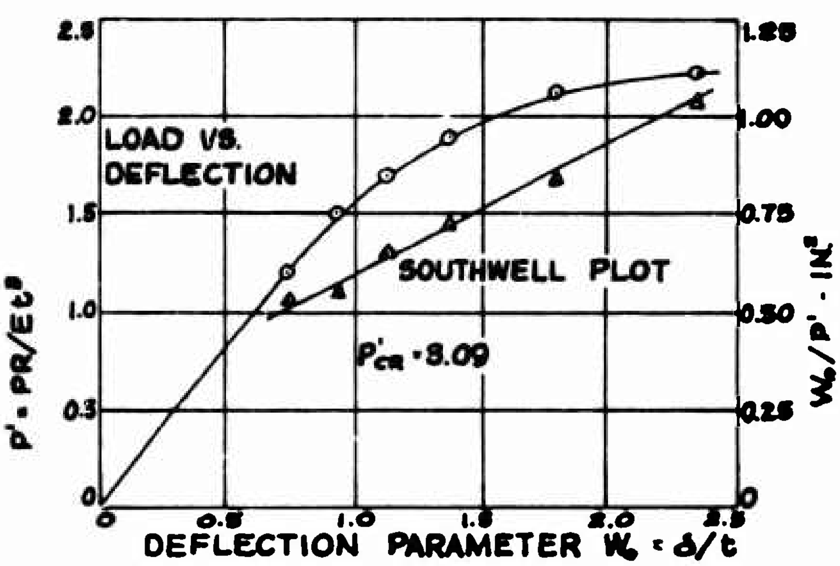

FIGURE 7. Load-Deflection Relationship and Corresponding Southwell Plot for Sphere with Point Load at Apex (Ref 19).

specimen with a λ of 6.4. The corresponding case for the Evan-Ivanowski, Cheng, and Loo research[3] (λ=6.3) is given in Figure 8. The critical values of $P'R/Et^3$ derived from the plots are in very close agreement, being 3.09 and 3.1 units, respectively.

Although the majority of the data cannot be analyzed in this fashion, it is interesting to note that those results which can be so treated appear to be consistent and to agree reasonably well with Ashwell's computed values. It would seem, however, that before a positive statement is made in this regard, another series of tests should be made with the method of correlating via load displacement firmly in mind from the onset. The behavior of a spherical cap under uniform external pressure has been considered analytically and experimentally by Kaplan and Fung.[21] The majority of their data is amenable to this method of analysis, and a typical δ/p versus δ plot is given in Figure 9. In Figure 10, the deduced critical pressures are plotted as a function of the geometric parameter λ, which describes the caps.

It is seen that all the points are on a smooth curve. The lower portion of this curve corresponds very well with that computed by Kaplan and Fung as an approximate solution to their equations.

The spherical cap under combined loading, a normal force at the apex and a distributed pressure, has been examined experimentally by Loo and Evan-Ivanowski.[22] Many of their load-displacement curves can be analyzed by the Southwell method, but there is no theory available for comparison.

The behavior of complete spheres under external pressure loading is another area in which problems of correlating test data currently exist.

An experimental study was made on a thin-walled nickel sphere to check the possibility of relating radial displacement, pressure, and theoretical critical external pressure. The experiment is reported in full in Reference 18. The basic material had a Young's modulus of 29.3×10^6 lb/in.^{2} as determined by routine material tests. This value was confirmed for several points on the sphere by measurement of decrease in radius as a function of pressure. The shell was elastically buckled;* when the point of buckle inception had been thus established, a test was made using a sensitive noncontacting displacement probe located at, or very near, the buckle center. The variation of displacement with pressure is clearly seen in Figure 11. There are two properties of this curve which are both apparent and important. First, the initial deflection is linear. Second, the final displacement curve is hyperbolic. The hyperbolic curve gives an excellent Southwell plot (Figure 12), whose slope corresponds closely

*The sphere was restrained against excessive motion by the use of an internal mandrel. As a result, on the second and subsequent buckling tests, the load levels for instability reached the same value as on the initial test.

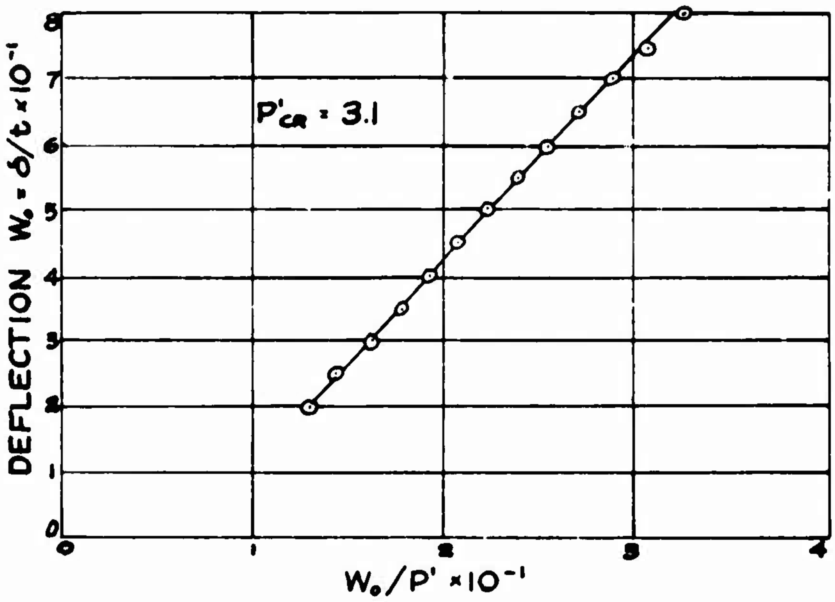

FIGURE 8. Southwell Plot for Sphere (Data from Ref 20).

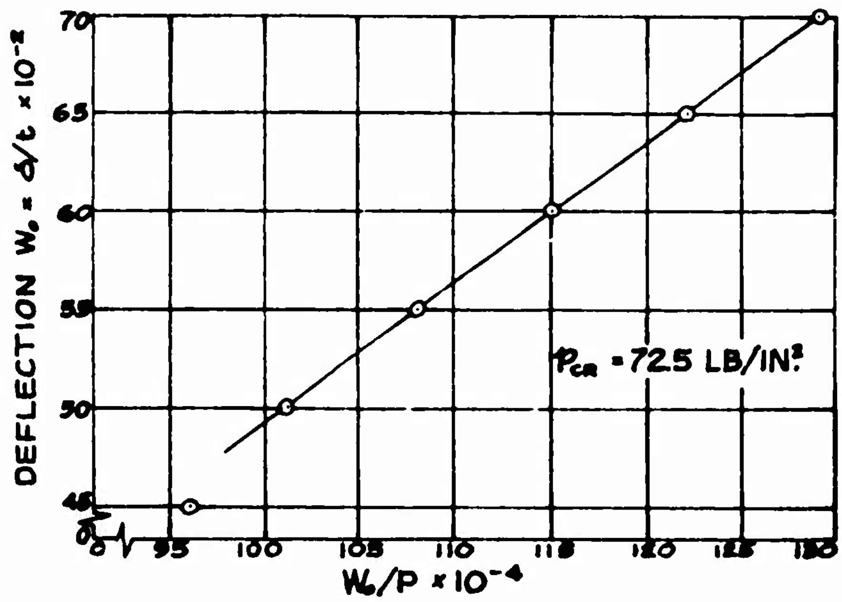

FIGURE 9. Southwell Plot for Spherical Cap under External Pressure (Ref 21).

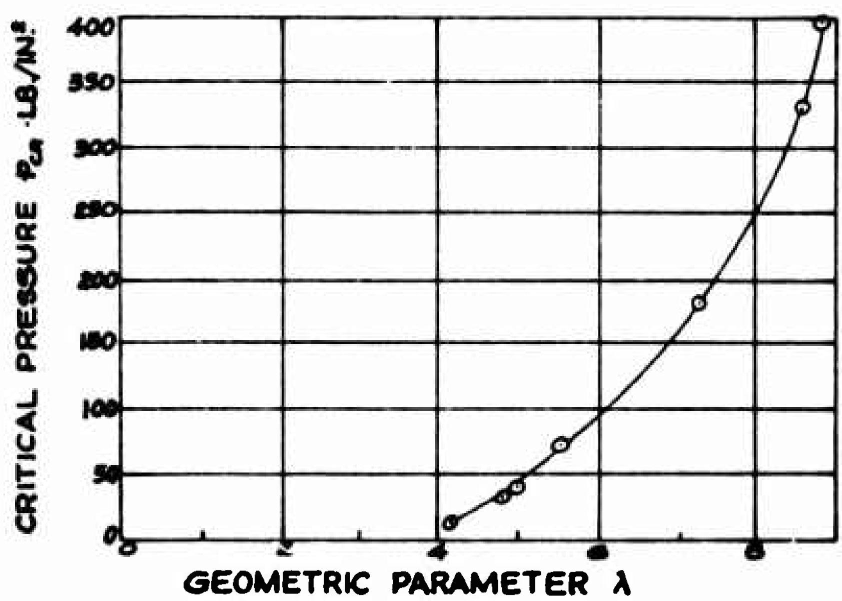

FIGURE 10. Critical Pressure Versus λ as Determined by Southwell Plots (ref 21).

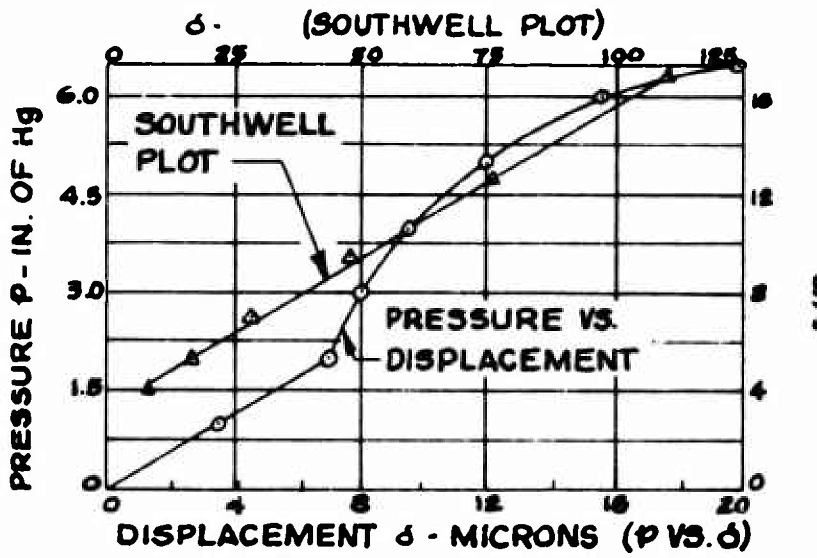

FIGURE 11. Pressure-Displacement Relation and Corresponding Southwell Plot for Nickel Sphere under External Pressure.

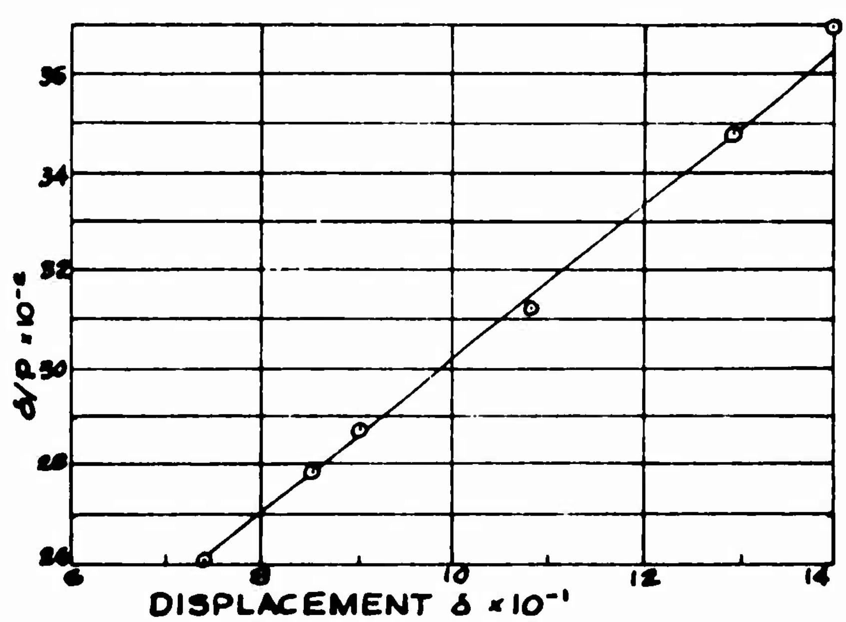

FIGURE 12. Southwell Plot for Sphere External Pressure (Data from Ref 23).

to the critical load computed on classic theory when the modulus has the value indicated by the initial slope.

The data presented by Thompson[23] have also been examined in the same manner.[12] The conclusions reached were identical. The Southwell plot predicts the classic critical load for this sphere, also. It should be noted that in this case, too, the buckle generating defect was located at a point which was characterized by an apparent reduced modulus. The modulus value used in the data correlation was that which corresponded to the initial linear portion of the load-displacement curve.

CONCLUSIONS

The experimental and theoretical study presented in this report concludes a broad analysis which has been conducted on the range of applicability of the Southwell plot as a means of data interpretation for tests on shells. The results presented, when taken in conjunction with those given in a previous report for plate structures, provide evidence that the method is of absolute generality. However, it may be reasoned in the case of structures in which many buckles can be formed, that the true process of buckling may be controlled by the individual character of the structures. In this case, imperfection could influence local boundary values and would thus show as an apparent lowering or an elevating in critical load as given by the Southwell plot. If this is so, we should anticipate that the distribution of Southwell values will be essentially normal if many observations are made at devious points over the body.

With this process, then, we feel confident in asserting that theories for perfect bodies which give estimates not in agreement with experimental values derived on the basis of Southwell-type plots must be considered inadequate, incomplete, or in error.

REFERENCES CITED

1. Nash, W. A., BIBLIOGRAPHY ON SHELLS AND SHELL-LIKE STRUCTURES, 1954-1956, University of Florida, Contract DA-01009-ORD-404, U. S. Army, Office of Ordnance Research, 1957.
2. Nash, W. A., BIBLIOGRAPHY ON SHELLS AND SHELL-LIKE STRUCTURES TMB Report 863, U. S. Navy Department, David Taylor Model Basin, Washington D. C., 1954.
3. Southwell, R. V., ON THE ANALYSIS OF EXPERIMENTAL OBSERVATIONS IN PROBLEMS OF ELASTIC STABILITY, Proceedings of the Royal Society, Vol. 135A, page 601.
4. Horton, W. H., Cundari, F. L., and Johnson, R. W., THE ANALYSIS OF EXPERIMENTAL DATA OBTAINED FROM STABILITY STUDIES ON ELASTIC COLUMN AND PLATE STRUCTURES, Israel Journal of Technology, Vol. 4, No. 1, 1967.
5. Westergaard, H. M., BUCKLING OF ELASTIC STRUCTURES, Transactions of American Society of Civil Engineers, Vol. 85, 1922, pages 576-654.
6. Langhaar, H. L., Boresi, A.P. Carver, D. R., ENERGY THEORY OF BUCKLING OF CIRCULAR ELASTIC RINGS AND ARCHES, Proceedings of Second U.S. National Congress on Applied Mechanics, 1954.
7. Sturm, R. G., A STUDY OF THE COLLAPSING PRESSURE OF THIN-WALLED CYLINDERS, Engineering Experiment Station Bulletin, No. 329, University of Illinois, Urbana, Illinois.
8. Cleaver, P. C., STRENGTH OF TUBES UNDER EXTERNAL PRESSURE, Royal Aircraft Establishment Report No. Structures 193, November 1955, Farnborough, Hants, England.
9. Galletly, G. D., and Reynolds, T. E., A SIMPLE EXTENSION OF SOUTHWELL'S METHOD FOR DETERMINING THE ELASTIC GENERAL INSTABILITY OF RING STIFFENED CYLINDERS SUBJECTED TO EXTERNAL PRESSURE, Proceedings SESA, Vol. XIII, No. 2, 1956, pages 141-151.
10. Kendrick, S., THE BUCKLING UNDER EXTERNAL PRESSURE OF CIRCULAR CYLINDRICAL SHELLS WITH EVENLY SPACED EQUAL STRENGTH CIRCULAR RING FRAMES, Part I NCRE/R No. 211, Naval Construction Research Establishment, Dumfurine, Scotland, 1953.
11. Donnell, L. H., ON THE APPLICATION OF SOUTHWELL'S METHOD FOR THE ANALYSIS OF BUCKLING TESTS, S. Timoshenko 60th Anniversary Volumn, MacMillan Company, New York, 1938.
12. Flugge, W., STRESSES IN SHELLS, Springer, Berlin, 1960.
13. Lundquist, E. E., GENERALIZED ANALYSIS OF EXPERIMENTAL OBSERVATIONS IN PROBLEMS OF ELASTIC STABILITY, NACA Technical Note 658, 1938.
14. Horton, W. H., Bailey, S. C., and Smith, S., A NEW FAMILY OF TEST MACHINES AND TRANSDUCERS FOR STABILITY STUDIES ON THIN-WALLED SHELLS, Paper in Preparation.
15. Horton, W. H., Craig, J. I., and Shover, D., ON THE MEASUREMENT OF MOTIONS, DEFORMATIONS AND GEOMETRIC PARAMETERS IN STUDIES OF INSTABILITY OF THIN-WALLED SHELLS, Paper in preparation.
16. Cundari, F. L., ON THE ANALYSIS OF EXPERIMENTAL DATA OBTAINED FROM STABILITY STUDIES, Ph. D. Thesis in preparation.
17. Banks, M., THE EFFECT OF FIBER DIRECTION ON THE STABILITY OF SINGLE-LAYER RESIN IMPREGNATED GLASS CLOTH CYLINDERS UNDER TORSION, Engineer's Thesis in preparation.
18. LeBouc, P., SOME STUDIES ON THE INSTABILITY OF SPHERES AND SPHERICAL CAPS, **Engineer's Thesis in preparation.**

19. Ashwell, D. G., ON THE LARGE DEFLECTION OF A SPHERICAL SHELL WITH AN INWARD POINT LOAD, Proceedings of the Symposium Theory of Thin Elastic Shells (Delft, 24th-28th August 1959), 1960.
20. Evan-Ivanowski, R. M., Cheng, H. S., and Loo, T. C., EXPERIMENTAL INVESTIGATIONS OF DEFORMATIONS AND STABILITY OF SPHERICAL SHELLS SUBJECTED TO CONCENTRATED LOAD AT THE APEX, Proceedings of the Fourth U.S. National Congress of Applied Mechanics, 1962, page 563.
21. Kaplan, A., and Fung, Y. C., A NONLINEAR THEORY OF BENDING AND BUCKLING OF THIN ELASTIC SHALLOW SPHERICAL SHELLS, NACA TN 3212, August 1954.
22. Loo, T. C., and Evan-Ivanowski, R. M., DEFORMATIONS AND COLLAPSE OF SPHERICAL DOMES SUBJECTED TO UNIFORM PRESSURE AND NORMAL CONCENTRATED LOADS AT THE APEX, Proceedings of World Conference on Shells and Structures, San Francisco, 1962.
23. Thompson, J. M. T., THE ELASTIC INSTABILITY OF A COMPLETE SPHERICAL SHELL, The Aeronautical Quarterly, Vol. 13, Part 2, May 1962, page 189.

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13. ABSTRACT

This report demonstrates, from experimental studies, that the classic critical loads computed from small displacement theory are correct for perfect shells under ideal loading conditions. The method used is an implicit rather than an explicit one. It makes use of the fact that for a realistic structure, the elastic deformation can be associated with the initial displacement from ideal form, the load which produces the motion, and the classic load for the structure by the hyperbolic expression

$$\delta\left(\frac{P_{cr}}{P} - 1\right) = \delta_o$$

By choosing the variables to be δ/P and δ, the relationship can be presented in the form of a straight line whose slope corresponds to the critical load for the ideal case. This is, in essence, the method developed by Aryton and Perry, in 1889, to analyze column data. The generalization to include other structures was foreseen by Southwell in his classic paper of 1932, although he offered no proof. In effect a general proof exists in the theory of elastic stability as presented by Westergaard in 1922. However, the practical application to shells has, until now, not been made.

By using this technique, the behavior of cylinders under the action of external pressure, torsion, and axial load and of the instability of spheres and spherical caps under the action of external pressure is examined. In the main, the analyses are conducted on experimental data already published. However, there are notable exceptions, the cylindrical shell under axial load a case in point.

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